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WADC TECHNICAL REPORT 52-76

(Uncl) NAVIGATIONAL SYSTEMS COMBINING INERTIAL AND GROUND VELOCITY VECTOR INFORMATION

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WADC TECHNICAL REPORT 52-76 [redacted]

NAVIGATIONAL SYSTEMS COMBINING INERTIAL AND GROUND VELOCITY VECTOR INFORMATION

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Flight Research Laboratory

April 1952

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Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

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FOREWORD

This report was prepared by the Flight Research Laboratory under an internal Project on Expenditure Order No. R-466-1-4. It was decided to investigate in detail the problem of damping and the response characteristics of systems combining inertial and ground velocity indication. The broad aspects of damping had been treated previously in AF Technical Report No. 6045 dated October 1949 entitled "Earth's Radius Pendulum." This work has been done under the direction of Dr. J. E. Clemens, Chief of Physics Research Branch, Flight Research Laboratory.

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ABSTRACT

This report deals with the possibilities of combining inertial and ground velocity vector information in order to obtain a system with improved overall response.

The classification of the title of this report is UNCLASSIFIED.

PUBLICATION REVIEW

Manuscript copy of this report has been reviewed and found satisfactory for publication.

FOR THE COMMANDING GENERAL:

Leslie B. Williams

LESLIE B. WILLIAMS
Lt. Colonel, USAF
Chief, Flight Research Laboratory
Research Division

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INTRODUCTION

Due to careful scientific research and skillful design certain inertial systems have achieved such a state of perfection that they have given excellent performance in a number of test flights. These test flights have proven the usefulness of those systems for long range navigation of airplanes and guided missiles.

Recently it has been claimed that another system based on the doppler radar can give the ground speed and the drift angle under flight conditions to such a degree of accuracy, that open loop integration of these values will lead to a position indication regarded as good enough on which to base a long range navigation system.

This claim led some of the sponsors of the inertial system development to the consideration it might be worthwhile to use the doppler radar as an additional source of information about the velocity vector. They figured that the platform damping could be increased as the velocity vector information improved.

It is the aim of the Chapters I and II of this report to deal with an inertial system with additional information concerning the velocity vector, and to deduce its transfer functions. Special emphasis is placed on the influence of the errors of the velocity vector information. However, the deductions in this report are not restricted to the case that the information concerning the ground velocity vector is delivered by a doppler radar; they are equally valid for any other source of information (e.g. systems based on optical or infra-red scanning etc.).

The sponsors of the doppler radar development intend to use a simple inertial platform with accelerometer in order to improve the short time indication which is otherwise

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disturbed by noise. This possibility is dealt with in Chapter III.

The long range accuracy of the combined system is determined in the first case only by the inertial platform, and in the second case only by the doppler radar. It is evident that the long range accuracy of the combined system cannot be better than the long range accuracy of the dominant subsystem.

CHAPTER 1

PROBLEM OF PLATFORM DAMPING

As is well known the damping of an inertial platform is a difficult problem. The platform should be damped against the true vertical. However, without additional information the apparent vertical is the only term available on the platform to damp against. It is evident that every change of the angle between the true and the apparent vertical produced by a change in acceleration (jerk) will disturb the platform through the damping. Thus one is faced with the dilemma of compromising between a smaller damping term than desirable, but having a low forced error disturbance, or a damping term of the proper value with a larger forced error disturbance.

The ground velocity vector is the additional information needed to compensate for the error introduced by the damping term. The authors proposed to use as compensation the indicated airspeed for the lack of the velocity vector[1]. Such a system is described by Hutzenlaub[2]. It is evident that the indicated airspeed, representing only the magnitude of the ground velocity vector falsified by the wind, is an imperfect substitute. The recent development of devices to measure the true ground velocity vector inspired the authors to this investigation.

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CHAPTER II

SPIRE SYSTEM WITH GROUND VELOCITY VECTOR METER

The MIT–Spire system (see Bibliography No. 2) shall be taken as an example of an inertial system. This system is aligned with respect to the preselected great circle course and the surface of the earth, and it is automatically controlled so that it retains this alignment all the time within very close limits. The sensing elements are two accelerometers; the one measuring the component tangential to the great circle course furnishes the input for the range computer, the other one measuring the component normal to the great circle course furnishes the input for the tracking computer. Thus it is evident that the tangential component of the velocity vector must be used to compensate for the disturbance introduced by the damping of the range computer, and the normal component of the velocity vector must be used to compensate for the disturbance introduced by the damping of the tracking computer.

Because the angle $A^*_{(P-path)}$ between the programmed great circle course and the path (see Fig. 2b) is always small, the tangential component is the true ground speed $V_{(E-A/C)}$, and the normal component is

$$V_{T(E-A/C)} = V_{(E-A/C)} \cdot A^*_{(P-path)} \qquad (1)$$

However, the angle $A^*_{(P-path)}$ is not measured directly, since the ground velocity vector meters give the ground speed $V_{(E-A/C)}$ and the drift angle $A^*_{(x-path)}$ of the path against the longitudinal axis X of the airplane.

To obtain the desired angle $A^*_{(P-path)}$ the angle $A^*_{(P-x)}$ of the longitudinal axis against the programmed great circle course which is always indicated without error on the Spire gimbal system must be added to $A^*_{(x-path)}$ (see Fig. 2b).

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$$A^*_{(P\text{ - }path)} = A^*_{(P\text{ - }x)} + A^*_{(x\text{ - }path)} \tag{2}$$

The velocity components $(M)\,V_{(E\text{ - }A/C)}$ and $(M)\,V_{T(E\text{ - }A/C)}$ which are furnished by the ground velocity vector meter obviously have some error. It is assumed that the indicated values can be represented as the sum of a correct term and an error term (EM).

Thus one obtains the equations

$$(M)\,V_{(E\text{ - }A/C)} = V_{(E\text{ - }A/C)} + (EM)\,V_{(E\text{ - }A/C)} \tag{3}$$

and

$$\begin{aligned}(M)\,V_{T(E\text{ - }A/C)} &= V_{T(E\text{ - }A/C)} + (EM)\,V_{T(E\text{ - }A/C)} \\ &= V_{(E\text{ - }A/C)} \cdot A^*_{(P\text{ - }path)} + \\ &\quad + A^*_{(P\text{ - }path)} \cdot (EM)\,V_{(E\text{ - }A/C)} + \\ &\quad + V_{(E\text{ - }A/C)} \cdot (EM)\,A_{(x\text{ - }path)}\end{aligned} \tag{4}$$

The term with the product of the two errors is neglected in (4), because it is a small second order term.

1. <u>Transfer Function of the Range Indicating System with Ground Velocity Vector Meter</u>

Fig. 1a shows the block diagram of the system with a damping loop and a ground velocity vector meter; which corresponds with the system Fig. V–A on page 143 of Hutzenlaub's report[2].

The external information about the velocity shall be disregarded for a moment. It is well known that such a platform, if it has no damping loop, is on the borderline of instability, but it is insensitive against external disturbances (i.e. against changes in acceleration). On the other hand, such a platform with a damping loop is sensitive against changes in acceleration.

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The disturbing damping loop consists in the feed back of the velocity. If therefore, as shown in Fig. 1a, an additional external source of information about the velocity is used to compensate for the disturbing internal velocity feed back, the platform behaves like an undamped one with respect to external disturbances, but nevertheless any oscillations of the platform (however they may have been originated) die out.

The transfer function can be derived from Fig. 1a. One obtains

$$A_{(P\,\cdot\,i)} = \frac{\frac{1}{W_E^2}s^3 + S_2 s^2 + s + S_3}{S_1 s^3 + S_2 s^2 + s + S_3}\, A_{(P\,\cdot\,t)} + \frac{\left(S_2 - \frac{S_3}{W_E^2}\right) s}{S_1 s^3 + S_2 s^2 + s + S_3} \cdot \frac{(EM)\, V_{(E\,\cdot\,A/C)}(s)}{R_E} \qquad (5)$$

The terms caused by the correct part $V_{(E\,\cdot\,A/C)} = R_E s\, A_{(P\,\cdot\,t)}$ of the ground velocity information are contained in the first expression of the right side of equation *(5)*. The second expression comprises only the influence of the ground velocity meter error.

If the platform is tuned to 84.4 minutes, i.e. if one has made

$$S_1 = \frac{1}{W_E^2} \qquad (6)$$

the transfer function of the first term is equal to one. The error of the indication may be defined in the usual manner.

$$(E)\, A_{(P\,\cdot\,i)} = A_{(P\,\cdot\,i)} - A_{(P\,\cdot\,t)} \qquad (7)$$

Thus one obtains by combining *(7)* with *(5)* for the tuned platform $(S_1 = 1/W_E^2)$

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$$(E)\,A_{(P\,\text{-}\,i)} = \frac{\{S_2 + \frac{S_3}{W_E^2}\}\, s}{\frac{1}{W_E^2}\; s^3 + S_2 s^2 + s + S_3} \cdot \frac{(EM)\,V_{(E\,\text{-}\,A/C)}\,(s)}{R_E} \qquad (8)$$

Finally we introduce the dimensionless operator

$$D = \frac{s}{W_E} \qquad (9)$$

and divide by $V_{(E\,\text{-}\,A/C)}/R_E$, obtaining

$$\frac{R_E(E)\,A_{(P\,\text{-}\,i)}}{V_{(E\,\text{-}\,A/C)}} = \frac{1}{W_E} \cdot \frac{\{S_2 - \frac{S_3}{W_E^2}\}\, W_E \cdot D}{D^3 + S_2 W_E D^2 + D + \frac{S_3}{W_E}} \cdot \frac{(EM)\,V_{(E\,\text{-}\,A/C)}(D)}{V_{(E\,\text{-}\,A/C)}} \qquad (10)$$

Comparing the transfer function (10) with the transfer function of the system without velocity meter information as it is written on page 154 of Hutzenlaub's report, one recognizes that the error is reduced to the value of the product of this error and the relative velocity error. That is indeed a worthwhile improvement.

It may be noted that the transfer function equation (10) is positive, while the transfer function for the system without velocity information is negative. This is evident when one bears in mind that the velocity meter information is added, while the feed back of the speed $S_2 s\, A_{(P\,\text{-}\,i)}$ is subtracted (see Fig. 1a).

2. Transfer Function of the Tracking System with Ground Velocity Vector Meter

Fig. 2a shows the block diagram of the system and Fig. 2b the nomenclature and kinematics. Investigating the tracking system, it is evident that the velocities, angles etc. are rectangular to the plane of the programmed great circle course. In the formulas this fact is expressed by the subscript T. (Concerning $V_{T(E\,\text{-}\,A/C)}$ see the formulas (1) to (4)).

The transfer function of the tracking computer with Schuler tuning (84.4 minutes

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period) for a cross wind disturbance can be derived from Fig. 2a.

$$\frac{R_E A_{T(P-t)}}{V_{T(E-air)}} = \frac{1}{W_E} \cdot F_1(D) - \frac{1}{W_E} F_2(D) \cdot F_3(D) \cdot \frac{(EM)\, V_{T(E-A/C)}(D)}{V_{T(E-air)}} \qquad (11)$$

with

$$F_1 = \frac{D}{(C_3 + 1) D^2 + \frac{C_1}{W_E} D + \frac{C_2}{W_E^2}}$$

$$F_2 = \frac{C_3 D^2 + \frac{C_1}{W_E} D + \frac{C_2}{W_E^2}}{(C_3 + 1) D^2 + \frac{C_1}{W_E} D + \frac{C_2}{W_E^2}}$$

$$F_3 = \frac{\left(S_2 - \frac{S_3}{W_E^2}\right) W_E D}{D^3 + S_2 W_E D^2 + D + \frac{S_3}{W_E}}$$

The first term of the right side of (11) is independent of the velocity vector meter error; F_1 represents the transfer function of the control computer. The characteristic equation (i.e. the denominator) describes a second order system with the mass term $(C_3 + 1)$, the damping term C_1/W_E and the spring term C_2/W_E^2; but one must have in mind that the equation is written in units of the dimensionless operator $D = \frac{s}{W_E}$ (which is the ratio between the operator s of the system and the Schuler frequency $W_E = 4.47$ hrs.$^{-1}$). In an earlier investigation[3] the authors found that a tracking computer without ground velocity vector information gives a response as

[redacted]

good as possible with C_3 = 10 to 20, $C_1 = 20.7\ hrs^{-1}$, $C_2 = 24.8\ hrs^{-2}$. The dimensionless natural frequency was 0.22 and the damping ratio 0.3.

The situation is much more favorable for the system with ground velocity vector meter because of the achieved decoupling. C_3 can be made zero, and from the viewpoint of stability there is no objection to change C_1 or C_2 in order to obtain a faster response and a better damping ratio. With $C_1 = 20.7\ hrs.^{-1}$ and $C_2 = 218\ hrs.^{-2}$ one obtains the dimensionless frequency 2.12, and the damping ratio 0.7.*

The second term of equation (11) represents the disturbance of the system due to the error of the ground velocity vector meter. It is represented by the product of two transfer functions $F_2(D)$ and $F_3(D)$. $F_2(D)$ contains the constants of the control computer and $F_3(D)$ those of the indication computer. $F_2(D)$ approaches 1 if $C_3 >> 1$. Then the characteristic equation of the second term is of the third order.** The condition for stability due to Routh's criterion is

$$S_2 W_E^2 > S_3 \qquad (12)$$

This can be fulfilled easily, because in the system with ground velocity vector meter there are no restrictions to the damping constant S_2.

If C_3 is chosen small (e.g. zero) the transfer function $F_2(D)$ is unequal to 1. Then the characteristic equation can be represented by the product of a quadratic and cubic equation. Because of the factorization the stability check for the fifth order system is

* The authors do not see any reason why almost 9 fold increase of the gain C_2 should lead into difficulties, because the noise is very low in this loop due to the manifold integration. Anyway, a large increase should at least be possible.

** The loop with S_3 was added to obtain the correct response on the low frequency side.

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simple; it is sufficient to show that both factors represent stable systems. Besides the requirement that all coefficients must be positive, the only additional requirement is the fulfillment of Routh's criterion for the cubic equation, represented in (18). This shows that the tracking computer with ground velocity vector meter can be made stable very easily.

3. Comparison Between the Tracking System Without and With Ground Velocity Vector Meter

At this point it might be interesting to compare the system without and the one with ground velocity vector meter in order to appreciate fully the enormous gain in stability of the tracking computer with ground velocity vector meter.

The earlier investigations conducted at M. I. T.[2] had revealed that a tracking system without ground velocity vector information, and with indication computer tuned to 84.4 minutes becomes dynamically unstable the more the gain S_2 of the damping loop is increased. The authors[3] showed that stability could be achieved by adding a damping loop C_3 to the control computer, but even in this case the response is not very good.

In Table 1a, page 9, are contrasted the coefficients of the characteristic equation of the system without and the one with ground velocity vector. Those coefficients which do not coincide in both cases are framed. The stability of the system can be deduced from the coefficients in Table 1a, page 9, by utilizing Routh's stability criterion. However, in the case of the fifth order systems this task is somewhat cumbersome. Therefore the expressions of Routh's stability criterion are written down for the simplified case without S_3 and C_3 in Table 1 b, page 9. This advantage of the system with doppler radar is illustrated by Fig. 3.

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CHARACTERISTIC EQUATIONS FOR TRACKING SYSTEM

With S_3 and C_3

$$a_5 D^5 + a_4 D^4 + a_3 D^3 + a_2 D^2 + a_1 D + a_0$$

	Without Doppler	With Doppler
a_5	1	1
a_4	$\frac{C_1}{W_E(C_3+1)} + \boxed{\frac{W_E S_2}{C_3+1} + \frac{S_3 C_3}{W_E(C_3+1)}}$	$\frac{C_1}{W_E(C_3+1)} + \boxed{W_E S_2}$
a_3	$1 + \frac{C_2}{W_E^2(C_3+1)} + \boxed{\frac{S_3 C_1}{W_E^2(C_3+1)}}$	$1 + \frac{C_2}{W_E^2(C_3+1)} + \boxed{\frac{S_2 C_1}{C_3+1}}$
a_2	$\frac{S_3}{W_E} + \frac{C_1}{W_E(C_2+1)} + \boxed{\frac{S_3 C_2}{W_E^3(C_3+1)}}$	$\frac{S_3}{W_E} + \frac{C_1}{W_E(C_3+1)} + \boxed{\frac{S_2 C_2}{W_E(C_3+1)}}$
a_1	$\frac{S_3 C_1}{W_E^2(C_3+1)} + \frac{C_2}{W_E^2(C_3+1)}$	$\frac{S_3 C_1}{W_E^2(C_3+1)} + \frac{C_2}{W_E^2(C_3+1)}$
a_0	$\frac{S_3 C_2}{W_E^3(C_3+1)}$	$\frac{S_3 C_2}{W_E^3(C_3+1)}$

Routh's Criterion: $(a_1 a_2 - a_0 a_3)(a_3 a_4 - a_2 a_5) > (a_1 a_4 - a_0 a_5)^2$

Table 1a

Without S_3 and C_3

$$a_4 D^4 + a_3 D^3 + a_2 D^2 + a_1 D + a_0$$

	Without Doppler	With Doppler
a_4	1	1
a_3	$\frac{C_1}{W_E} + W_E S_2$	$\frac{C_1}{W_E} + W_E S_2$
a_2	$1 + \frac{C_2}{W_E^2}$	$1 + \frac{C_2}{W_E^2} + \boxed{S_2 C_1}$
a_1	$\frac{C_1}{W_E}$	$\frac{C_1}{W_E} + \boxed{\frac{S_2 C_2}{W_E}}$
a_0	$\frac{C_2}{W_E^2}$	$\frac{C_2}{W_E^2}$

Routh' Criterion: $(a_1 a_2 - a_0 a_3)\, a_3 > a_1^2 a_4$

Without Doppler: $W_E^2 C_1 > W_E^2 S_2 C_2 + C_1 C_2$

With Doppler: $W_E^2 \{S_2 C_1 + S_2^2 C_2 + 1\} + S_2 C_1 C_2 + C_1^2 + C_2^2 > 2C_2$

Table 1b

[redacted]

[redacted]

CHAPTER III

DOPPLER RADAR WITH INERTIAL INFORMATION TO IMPROVE THE SHORT TIME INDICATION

As will be pointed out in Chapter IV, the short time indication of the doppler radar is disturbed by noise. One can filter out the noise, but then the response of the system becomes sluggish, as can be seen from Figure 4b, curve 2 which shows the response to a step input in acceleration. But if the area represented by the difference between the curves 1 and 2 of Figure 4b could be added, the response of this improved system would be correct. Indeed, the necessary quantity (see curve 3 of Figure 4b) can be formed from the indication of an accelerometer mounted on a horizontal platform.

The block diagram of this system is shown in Figure 5. The first order filter smooths the doppler velocity indication; the lag is compensated for by the accelerometer feed in. The transfer function can be easily deduced from Figure 5:

$$V_i = \frac{1}{s}\{CV + C(EDo)V - CV_i + a + (E)a\} \tag{13}$$

Having in mind that $a = sV$ one obtains:

$$V_i = V + \frac{C}{s+C}(EDo)V + \frac{1}{s+C}(E)a \tag{14}$$

This equation shows that the indication is correct at every instant, except for the errors introduced by the doppler radar and the indication of the acceleration. It can be shown that the latter error is essentially generated by the misalignment of the platform on which the accelerometers are fixed. If the angle $A(t-i)$ between the true vertical and the vertical indicated by the platform is small, one obtains:

$$(E)a = gA_{(t-i)} \tag{15}$$

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1. The Platform of the Doppler Radar

In the previous paragraph the platform with the accelerometers was assumed to be held horizontally, but no consideration was given to the fact how this could be achieved, In this paragraph shall be shown how the combined system consisting of the doppler radar and the platform can be linked together.

The sponsors of the doppler radar suggested, if one could use a simple first order platform (similar e.g. to the normal airplane horizon) as a base for the accelerometers, since a complete second order platform tuned to 84.4 min. means a very high expenditure for the purpose in question. As is well known the first order platform develops two kinds of errors: it tries to adjust itself into the apparent vertical during periods of accelerated flight, and it develops misalignment as a consequence of the velocity of the vehicle*. The latter error orignates from the fact that even during a uniform motion along the surface of the earth a misalignment of the platform must develop in order to generate the necessary error signal which makes the platform turn with the rate of the true vertical. Since the doppler radar gives information about the velocity vector one has in the combined system the possibility to compensate for the velocity error of the first order platform.

2. Transfer Functions of the Doppler Radar with a First Order Platform

Figure 6a shows the schematic and Figure 6b the block diagram of the combined system. The lower part of Figure 6b representing the doppler radar with low pass filter is identical with Figure 5. The upper part of Figure 6b is the complete representation of the first order platform with accelerometer and additional velocity feed in.

Two transfer functions of the system are of special interest, the transfer function which relates the indicated velocity to the true one, and the transfer function which relates the misalignment of the platform to the angular acceleration.

For the velocity transfer function one reads from Figure 6b the following four equations.

* and the rotation of the earth [redacted]

[redacted]

$$\{s + C_1\} V_i = C_1 V + C_1 (EDo) V + K_2 R_E A_{(i-a)} + K_2 R_E (E) A \tag{16}$$

$$A_{(i-a)} = A_{(o-a)} - A_{(o-i)} \tag{17}$$

$$A_{(o-a)} = \frac{1}{R_E s} \cdot \frac{s^2 + W_E^2}{W_E^2}, \tag{18}$$

$$A_{(o-i)} = \frac{K_1 A_{(i-a)}}{s} + \frac{K_1}{s}(E)A + \frac{C_2}{R_E s} V_i \tag{19}$$

$(EDo)\,V$ is the error of the doppler radar and $(E)A$ is an angular error in the alignment of the platform which might be superimposed to the misalignment of the platform due to the dynamics. This additional error $(E)A$ will be caused e.g. by Coulomb friction in the gimbal system etc., thus in most cases it will be a region of uncertainty.

This uncertainty in the platform alignment causes an uncertainty in the indication of the acceleration. Both are related by the equation:

$$(E)A = \frac{(E)a}{g} \tag{20}$$

Combining the equations (16) to (19) leads to:

$$V_i = \frac{\frac{K_2}{W_E^2} s^2 + C_1 s + (C_1 K_1 + K_2)}{s^2 + (C_1 + K_1)s + (C_1 K_1 + C_2 K_2)} V + \frac{C_1(s + K_1)(EDo)V + K_2 R_E s (E) A}{s^2 + (C_1 + K_1)s + (C_1 K_1 + C_2 K_2)} \tag{21}$$

The transfer function which relates V_i to V should become unity. First of all, this condition must be fulfilled for the steady state condition is

$$C_2 = 1 \tag{22}$$

The additional requirement to make the transfer function equal to one on the very high end of the frequency band $(s \rightarrow \infty)$ is evidently

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$$K_2 = W_E^2 \tag{23}$$

But that means, as shall be shown later, in Chapter III.3, the tuning of the whole system to the Schuler frequency.

One can make the transfer function equal to unity for all values of s by fulfilling the additional condition

$$K_1 = 0 \tag{24}$$

Also the meaning of this will become clear later.

Now the transfer function which relates the misalignment $A_{(t-i)}$ of the platform to the angular acceleration sV/R_E shall be deduced.

$$A_{(t-i)} = A_{(o-i)} - A_{(o-t)} \tag{25}$$

Using the relation

$$A_{(o-t)} = \frac{1}{s}\,\frac{V}{R_E} \tag{26}$$

in addition to the equations (17), (18), (19), (21) and to the steady state condition (22), one can deduce

$$A_{(t-i)} = \frac{1}{W_E^2} \cdot \frac{K_1 s + (C_1K_1 + K_2 - W_E^2}{s^2 + (C_1 + K_1)s + (C_1K_1 + K_2)} \cdot \frac{sV}{R_E} + \frac{C_1(EDo)V/R_E + \{K_1 s + (C_1K_1 + K_2)\}(E)A}{s^2 + (C_1 + K_1)s + (C_1K_1 + K_2)} \tag{27}$$

For the tuned system (i.e. $K_2 = W_E^2$) one obtains

$$A^*_{(t-i)} = \frac{1}{W_E^2} \cdot \frac{K_1 s + C_1K_1}{s^2 + (C_1 + K_1)s + (C_1K_1 + W_E^2)} \cdot \frac{sV}{R_E} + \frac{C_1(EDo)V/R_E + \{K_1 s + (C_1K_1 + W_E^2)\}(E)A}{s^2 + (C_1 + K_1)s + (C_1K_1 + W_E^2)} \tag{28}$$

And if finally is made $K_1 = 0$, the first right hand term vanishes completely, and the misalignment is caused only by the errors.

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$$A^{**}_{(t-i)} = \frac{C_1(EDo)V/R_B + W_B^2(E)\,A}{s^2 + C_1 s + W_B^2} \tag{29}$$

In conclusion the free oscillation of the system shall be studied. It is described by the denominator of equation (27), or of equation (21) and considering the steady state condition (22). The general characteristic equation of a second order system is

$$s^2 + 2RW_o s + W_o^2 = 0 \tag{30}$$

W_o means the frequency of the undamped system, and R is the damping ratio. As well known $R < 1$ characterizes a periodic motion, and $R \geq 1$ a periodic one. Comparing the coefficients of the denominator of (27) or of (21) with (22) with those of (30) one finds

$$R = \frac{1}{2} \cdot \frac{C_1 + K_1}{\sqrt{C_1 K_1 + K_2}} \tag{31}$$

$$W_o = \sqrt{C_1 K_1 + K_2} \tag{32}$$

Therefore, the aperiodic case will exist, if

$$4K_2 \leq (C_1 - K_1)^2 \tag{33}$$

or for the tuned system ($K_2 = W_B^2$, see equ. (23)), if

$$2\,W_B \leq |C_1 - K_1| \tag{34}$$

In order to obtain a good transient response K_1 should be made small compared with C_1. Since on the other hand the constant C_1 of the doppler radar filter is very large compared with W_B, the combined system will be always very strongly overdamped, if it is designed to have a good transient response.

Fig. 7 illustrates the transient response of the system for a step input in velocity. As can be readily seen from these curves, the transient response can be improved if K_1 is chosen small, i.e. if the time constant of the inertial platform is long as compared to the time constant

[redacted]

of the doppler filter. This requirement calls for high quality inertial components.

The curves in Fig. 7 refer to the principle part of the transfer function (21), describing the dynamic response of the system. The effects of the errors in ground velocity and acceleration measurements (εdo) and $(\varepsilon)\,a$, which are statistical in nature, have to be superimposed upon these response curves. The RMS – error in V_{ind} due to doppler error $(\varepsilon do)V$ with the filtering time used in the examples of Fig. 7 (τ = 20 sec) is less than 0.1 % (this statement is based on the results of F. B. Berger, reference 4). The RMS – error in V_{ind} due to acceleration $(\varepsilon)\,a$ will depend on the quality of the inertial components used.

3. The Combined System from Another Aspect

Now the reason for the behaviour of the combined system, as it was represented by equations (21), and (27) to (29), shall be investigated. For this purpose the system is shown in Figure 8 in two different representations. Figure 8a is an identical replot of Figure 6b, while Figure 8b shows the same block diagram in another arrangement. As one can see all the loops are completely alike in both figures. But Figure 8b reveals at once that this combination of the two first order systems (which was proposed as a simple arrangement in the beginning of this paragraph) is essentially identical with a damped second order platform with doppler velocity information.* It is evident that this second order platform can give the correct response during a transient state only, if tuned to the Schuler frequency. One recognizes further that the system has two damping loops, a feed back loop with the gain C_1 and a forward loop with the gain K_1. The disturbance of the system by the feed back loop is compensated by the doppler

* This results from the fact that two crossfeeds have been introduced.

radar velocity information, but the disturbance of the system by the forward loop is not compensated for. Therefore K_1 must be made zero, if the system shall be unsensitive against external disturbances. But by omitting K_1 the whole principle of the simple platform has been abandoned, because the second order platform with 84.4 minutes period requires components of the highest accuracy.

4. Combined System with Short Period 2nd Order Platform

Since one could not obtain a correct transient response with an essentially first order system (i.e. a system considerably influenced by K_1), the question arises, if one would have better chances with a non-tuned short period second order platform. Indeed, this is the case. Also in this case the overall system, due to the cross-feeds, as one loop which must be tuned to 84.4 minutes period. As is well known, this is the indispensable requirement for each system with correct transient response.

The system is shown in Figures 9 and 10. The additional loops are inclosed in a dotted box. If everything contained in the dotted box is omitted, Firgures 9 and 10 becomes identical with Figures 6 and 8. The complete system represented in Figure 10 has the advantage that the disturbing influence of K_1 on the transient response can be compensated for by the proper choice of the other constants (gains).

The transfer function can be deduced in the same way as it was done in paragraph III.2. Thus one finds for a system with the tuning

$$\begin{aligned} K_2 &= W_E^2 \\ C_2 &= K_3 \\ C_3 &= 1 - \frac{K_1}{W_E^2} \end{aligned} \tag{35}$$

the equation

$$V_i = V + \frac{\{s^2 + K_1K_3 s + K_1\}\, C_1\, (EDo)\, V + s\, \{s + K_1K_3\}\, (E)\, a}{s^3 + (C_1 + K_1K_3)\, s^2 + (C_1K_1K_3 + W_B^2)\, s + (C_1K_1 + W_B^2K_1K_3)} \tag{36}$$

The transfer function for the angle of the misalignment could also be deduced, but it shall be omitted here; however, it may be mentioned that the main term becomes equal to zero, if equations (35) are fulfilled. That is evident, because equation (36) can be valid only if the misalignment of the platform is equal to zero (except for the errors). The overall system described by (36) will be stable if

$$K_1 < W_B^2 + K_1K_3\,(C_1 + K_3) \tag{37}$$

This condition imposes certain restrictions on the selection of system parameters. If, for instance, a short period inertial platform shall be used then equation (37) requires that K_3 (a minimum platform damping term) have the form

$$K_3 > -\frac{C_1}{2} \pm \sqrt{(1 - \frac{W_B^2}{K_1} + \frac{C_1^2}{4}} \tag{38}$$

This is required to render overall system stability. Thus, positive as well as negative platform damping will make the combined system stable. However, the negative values for K_3 have no real significance because it would be very impractical to use a highly suitable inertial component in the overall system.

A final remark shall be made concerning the required accuracy of the system. It is evident that, in order to make the 84.4 minutes loop operate correctly, one needs very high grade components. If however the components are of a somewhat lower grade, the accuracy of the system deteriorates somewhat. But this shows up only in the error term of (36), the main transfer function remains essentially equal to unity.

CHAPTER IV

(APPENDIX)

MODE OF OPERATION AND ERRORS OF THE DOPPLER RADAR

The speed measurement with the doppler radar AN/APN-66 designed by General Precision Laboratory is based on the doppler frequency shift of radar pulses reflected from the ground. The device has an array of four antennae, two of which are looking right and left forward, and the other two are looking right and left aft. The forward and aft frequencies are combined crosswise. In this way one obtains and error signal if the antenna array is not aligned with the direction of the ground path. That error signal is used to servo the antenna array into the direction of the ground path.

Thus, the ground velocity vector is indicated in polar coordinates, i.e. the doppler radar gives the magnitude (ground speed) and the angle of the ground velocity vector against airplane structure, or against any other reference direction which can be uniquely related to airplane structure.

The speed indication of the doppler radar may be written:

$$(Do)\, V_{(E\,-\,A/C)} = V_{(E\,-\,A/C)} + (EDo)\, V_{(E\,-\,A/C)}(t) \qquad (39)$$

where $V_{(E\,-\,A/C)}$ *Speed of the aircraft against earth (true ground speed)*

$(EDo)\, V_{(E\,-\,A/C)}(t)$ *Error of doppler speed indication*

The error is composed of a constant bias and a random fluctuation (noise). The bias is claimed to be very small if a careful calibration is made by using a sufficiently long distance to average out the fluctuations. However, if the aircraft is flying over water, the indication of the ground speed will be falsified by ocean currents and may be influenced by wave motions.

The doppler noise is caused by the vibrations of the antenna array and by the changes in terrain and the consequent changes in the reflection of the earth's surface. If $(\varepsilon do)\,V(t)$ is a record of the doppler noise taken over a time interval $-T \leq t \leq +T$ then the Fourier transform of this record can be written as:

$$(EDo)\,V(\omega) = \mathfrak{F}\left[(\varepsilon do)V(t)\right] = \int_{-T}^{+T} (\varepsilon do)\,V(t)\,e^{-2\pi i \omega t}\,dt \tag{40}$$

The spectral density of the noise is defined as:

$$G_{(EDo)V}(\omega) = \lim_{T\to\infty} \frac{1}{T} \left|(EDo)\,V(\omega)\right|^2 \tag{41}$$

Equation (41) can be approximated sufficiently well by taking T large and omitting the limiting process.

Based on the spectral density (41) the RMS – error of velocity indication can be computed. If it is assumed that $(\varepsilon do)\,V(t)$ is not cross-correlated to $V(t)$, the influence of the doppler noise can be investigated independently from the dynamic system response. Designating the weighting function for the noise (i.e. the filter response function for unit impulse input) by $W(t)$ and defining

$$(\varepsilon)\,V_{ind}(t) = \int_{0}^{t} (\varepsilon do)V(t)\cdot W(t-\tau)\,d\tau \tag{42}$$

the noise transfer function $Y(s)$ is found by taking the Laplace transform of (42) and applying the real convolution theorem:

[redacted]

$$Y(s) = \mathcal{L}\,[W(t)] = \int_0^\infty W(t)e^{-st}dt = \frac{(E)V_{ind}(s)}{(EDo)V(s)} \tag{43}$$

The steady state component of the noise transfer function is obtained from equation (43) by putting $s = 2\pi i\omega$, i.e. by substituting the Fourier transform for the Laplace transform. Therefore:

$$Y(2\pi i\omega) = \mathfrak{F}\,[W(t)] = \int_0^\infty W(t)e^{-2\pi i\omega t}dt \tag{44}$$

The spectral density of the velocity indication error is then found to be:

$$G_{(E)V_{ind}}(\omega) = |Y(2\pi i\omega)|^2 \cdot G_{(EDo)V}(\omega) \tag{45}$$

And the RMS – error in V_{ind} can finally be computed from:

$$\left\{\overline{[(E)V_{ind}]^2}\right\}^{1/2} = \left\{\int_0^\infty G_{(E)V_{ind}}(\omega)\,d\omega\right\}^{1/2} = \left\{\int_0^\infty |Y(2\pi i\omega)|^2 \cdot G_{(EDo)V}(\omega)\,d\omega\right\}^{1/2} \tag{46}$$

A similar derivation holds for the statistical error of the angle $(\varepsilon do)\,\psi(t)$ of the ground velocity vector against airplane structure (also called the drift or crab angle).

Since noise records for $(\varepsilon do)V(t)$ and $(\varepsilon do)\,\psi(t)$ were not available to the authors, no numerical computations could be made. However, some conclusions regarding the RMS – error in V_{ind} can be drawn from theoretical results obtained by F. B. Berger in " Random Errors in Doppler Systems, " reference 4, where a smoothing time of 18 sec was found to be sufficient to reduce the RMS – error in doppler speed to less than 0.1% of true ground speed. Also the RMS – error in true ground heading was computed to be less than 0.1 degrees after 6 sec smoothing time. (Both results were

[redacted]

[redacted]

obtained for $V = 200$ mph). Since the inertial platforms usually have much longer time constants or periods, resp., it is anticipated that they will act as very adequate filters for the two components $(\varepsilon do)V(t)$ and $(\varepsilon do)\psi(t)$ of the doppler indication, so that the resulting RMS – error in V_{ind} is expected to be much less than 0.1% in magnitude and 0.1 degree in direction. For the 84.4 minute system the doppler noise should be of no importance.

[redacted]

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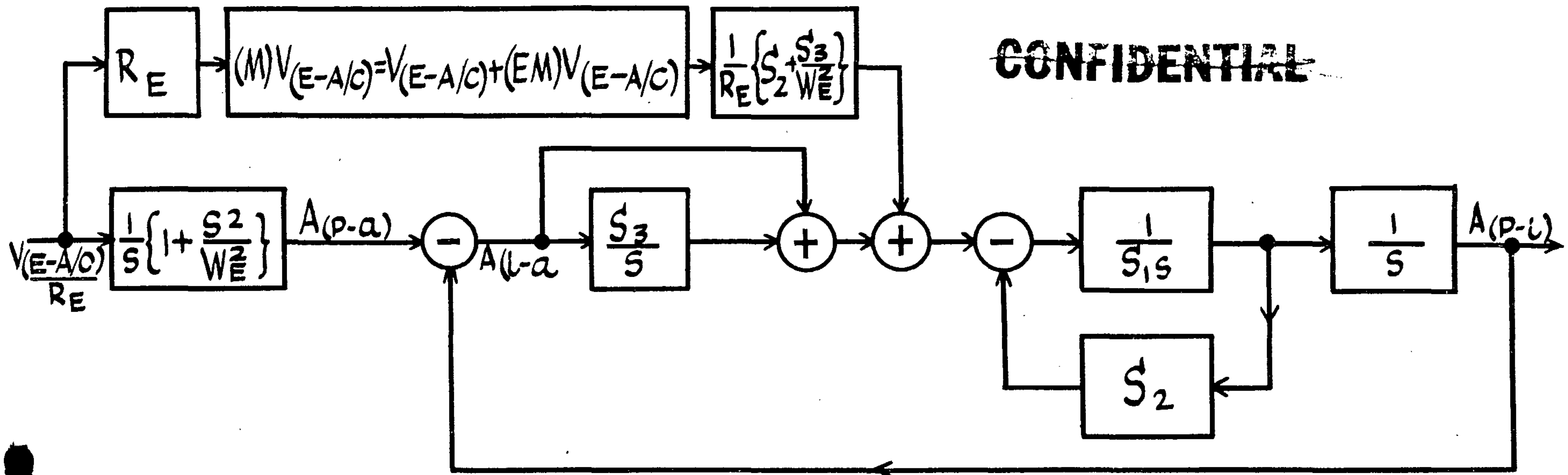

FIG. 1a BLOCK DIAGRAM

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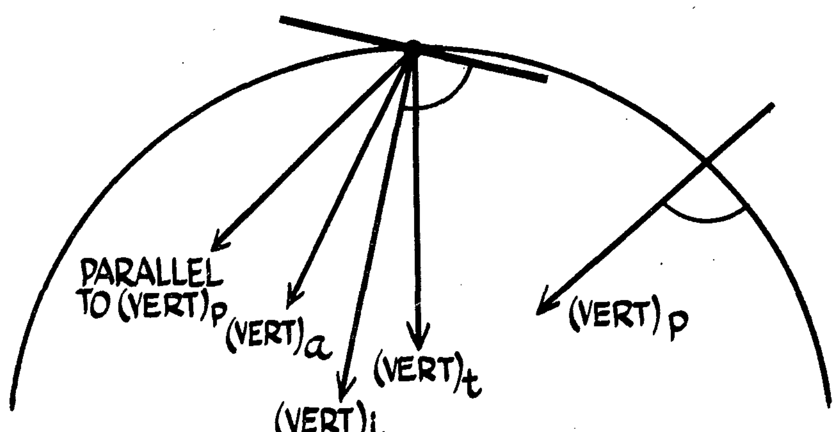

FIG. 1b NOMENCLATURE & KINEMATICS

- *KINEMATICS*

$$A_{(p-a)} = A_{(p-t)} - A_{(a-t)}$$

$$a_{A/C} = -R_E\{S^2 A_{(p-t)}\}$$

- *WITH SMALL ANGLE APPROXIMATION*

$$A_{(a-t)} = \frac{a_{A/C}}{g}$$

$$A_{(p-a)} = \left\{1 + \frac{S^2}{g/R_E}\right\} A_{(p-t)}$$

$$= \left\{1 + \frac{S^2}{W_E^2}\right\} A_{(p-t)}$$

FIG.1 SPIRE RANGE CONTROL SYSTEM WITH VELOCITY VECTORMETER

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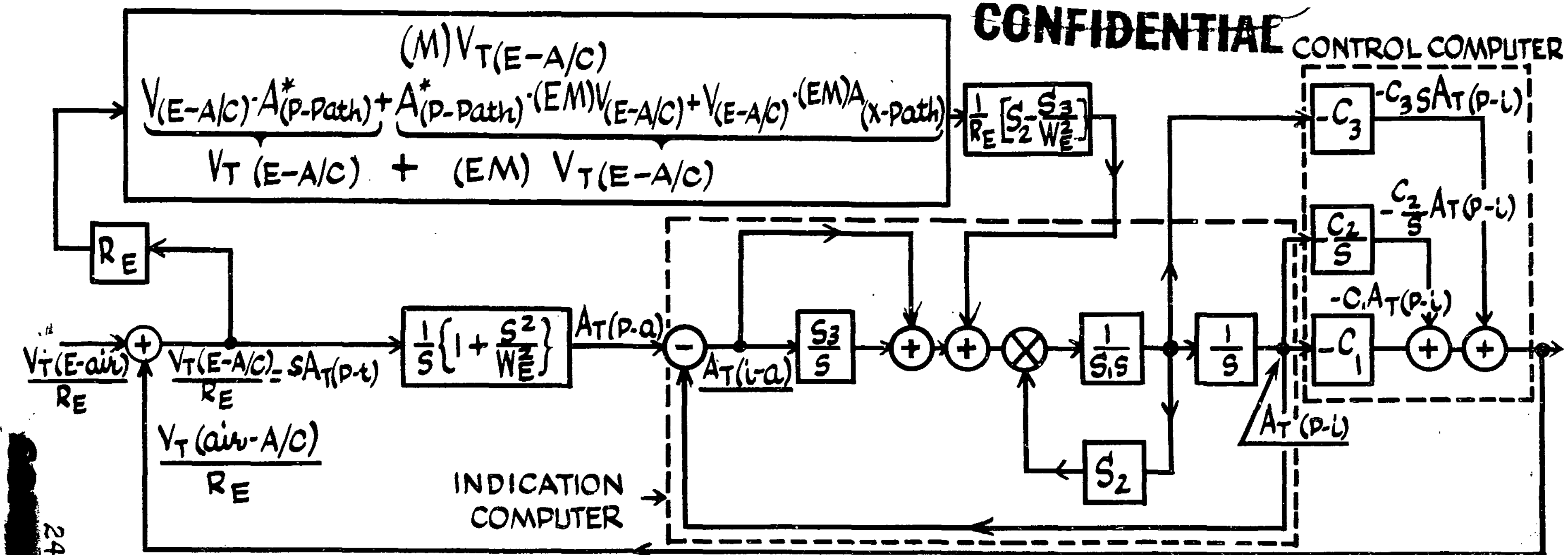
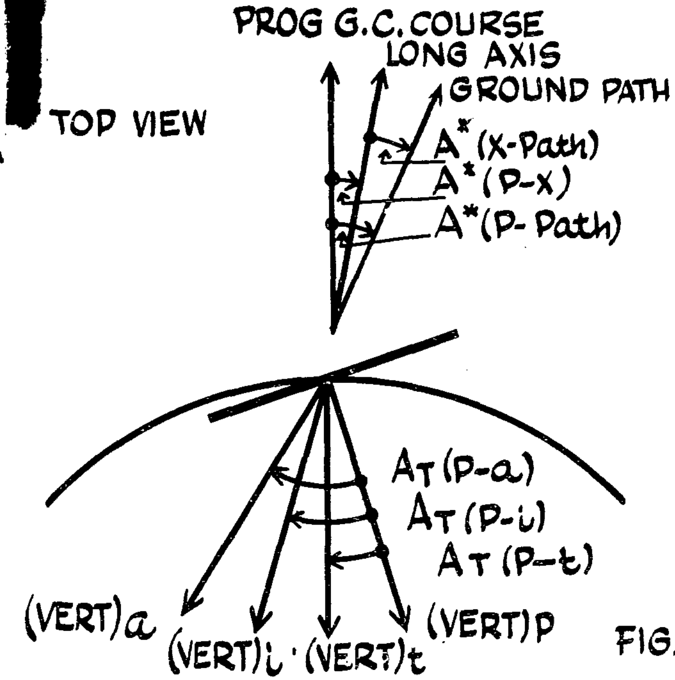


FIG. 2a BLOCK DIAGRAM



- ***KINEMATICS:***

$$A_T(P\text{-}a) = A_T(P\text{-}t) + A_T(t\text{-}a)$$

$$a_T(A/C) = R_E\{S^2 A_T(P\text{-}t)\}$$

- *WITH SMALL ANGLE APPROXIMATION*

$$A_T(t\text{-}a) = \frac{a_T(A/C)}{g}$$

$$A_T(P\text{-}a) = \left\{1 + \frac{S^2}{\frac{g}{R_E}}\right\} A_T(P\text{-}t)$$

$$= \left\{1 + \frac{S^2}{W_E^2}\right\} A_T(P\text{-}t)$$

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FIG. 2b NOMENCLATURE & KINEMATICS

FIG. 2 SPIRE TRACK CONTROL SYSTEM WITH VELOCITY METER DAMPING

[redacted]

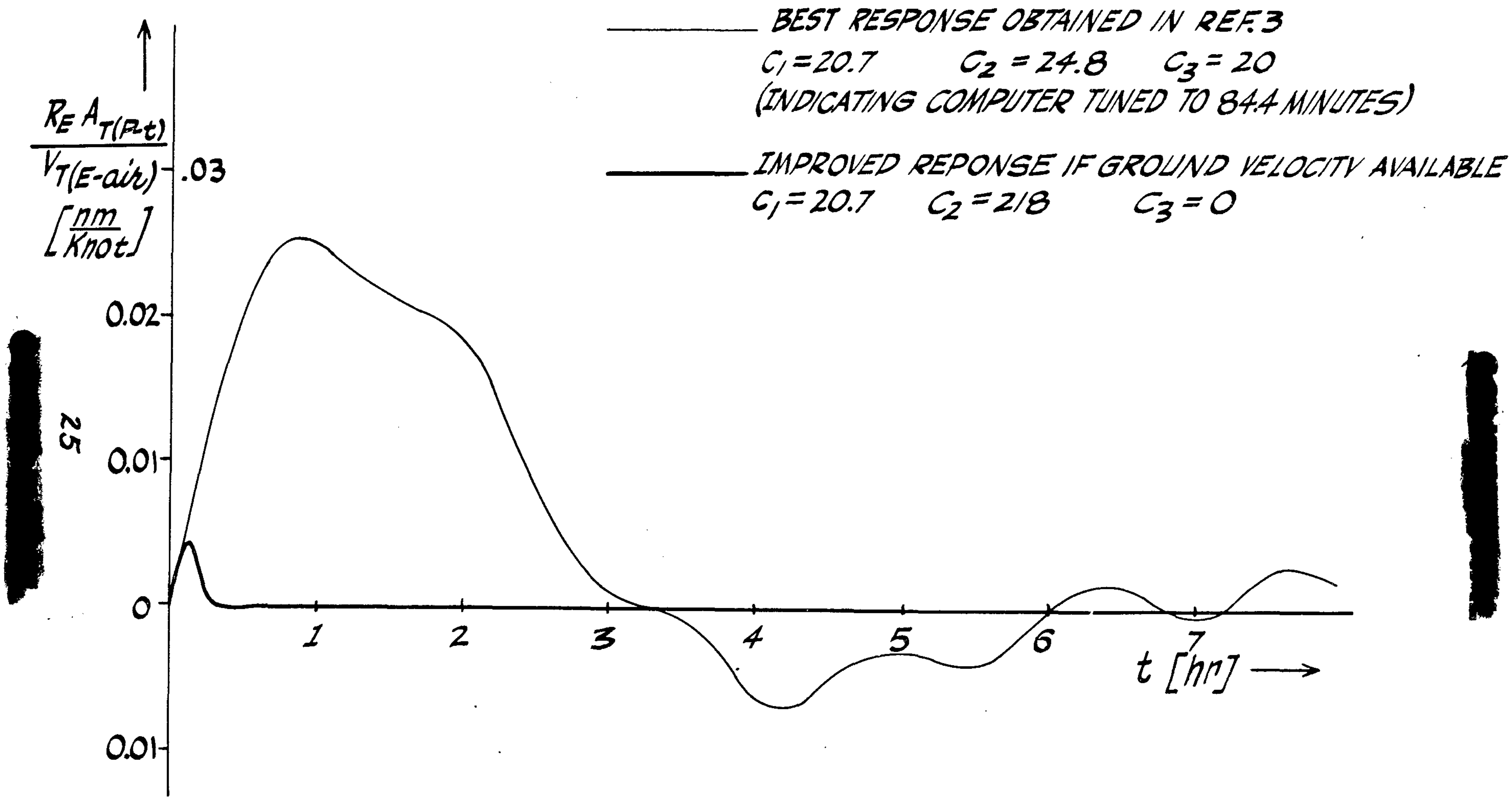

FIG. 3. TRANSIENT RESPONSE OF TRACKING CONTROL SYSTEM WITHOUT AND WITH GROUND VELOCITY INFORMATION

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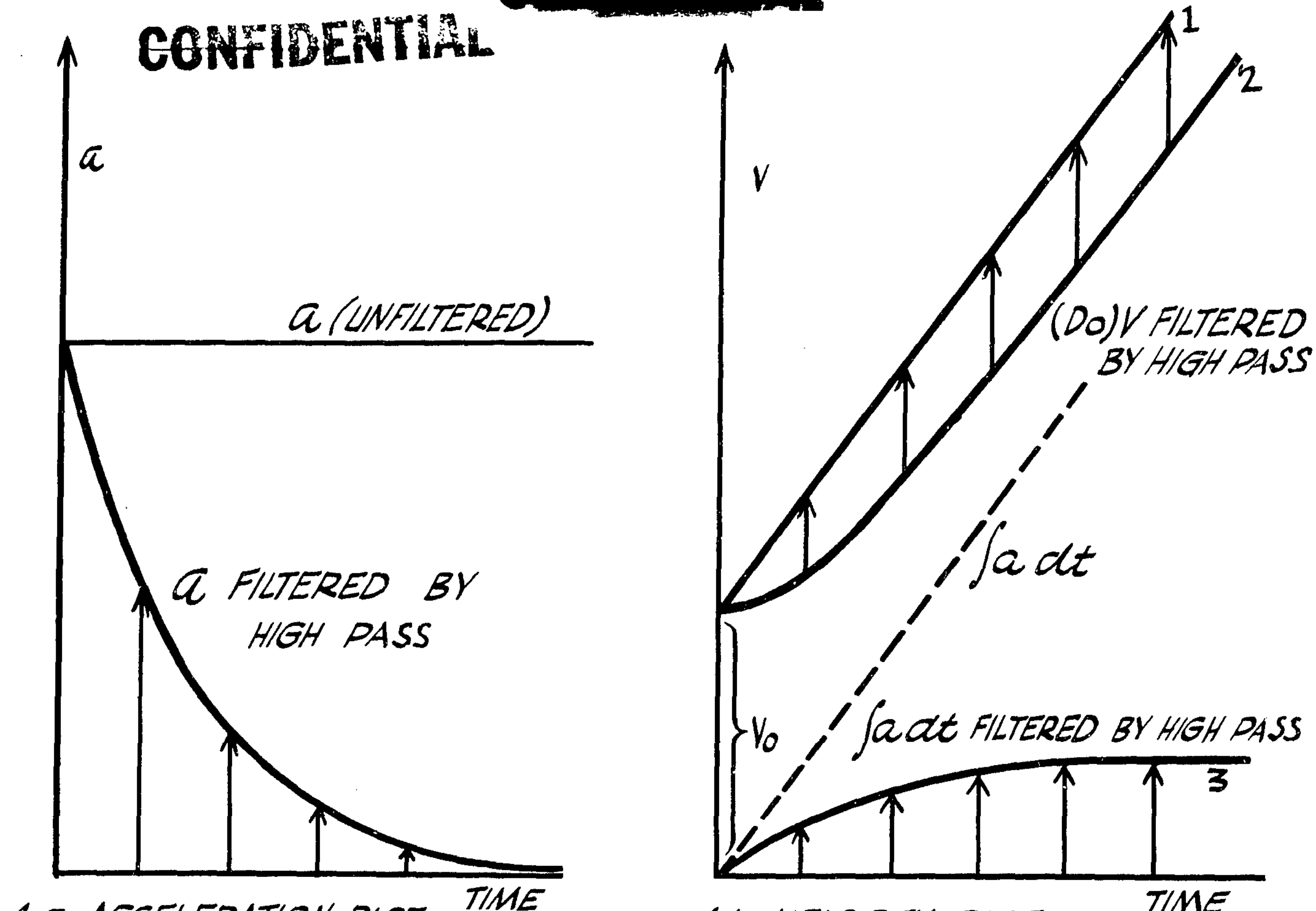

4a. ACCELERATION PLOT

4b. VELOCITY PLOT

FIG.4 FILTERED VELOCITY WITH ADDED INFORMATION FROM AN ACCELEROMETER

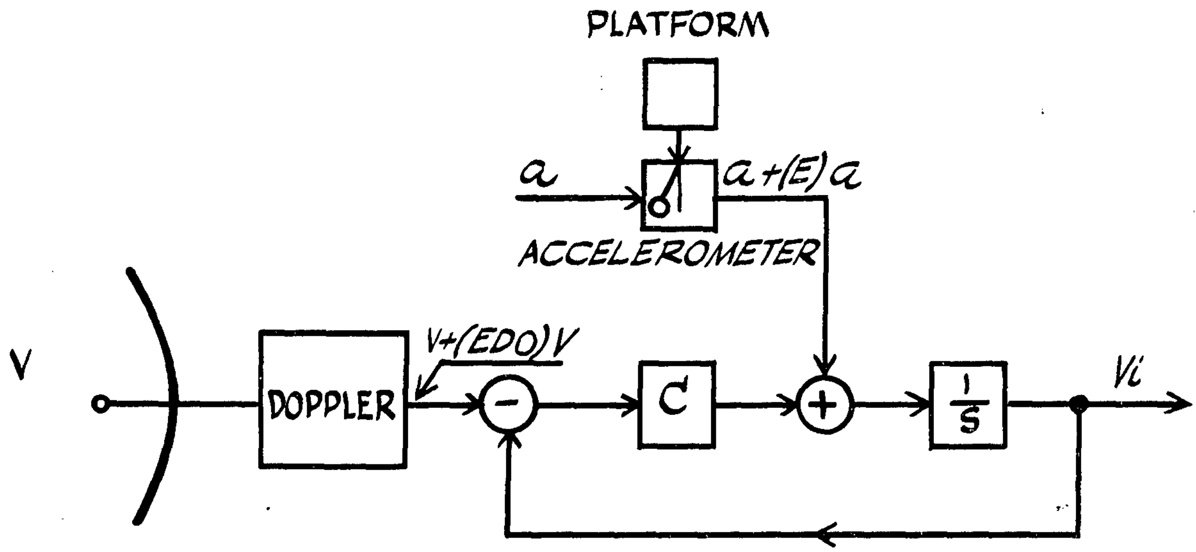

FIG.5 FILTERED DOPPLED WITH ACCELEROMETER

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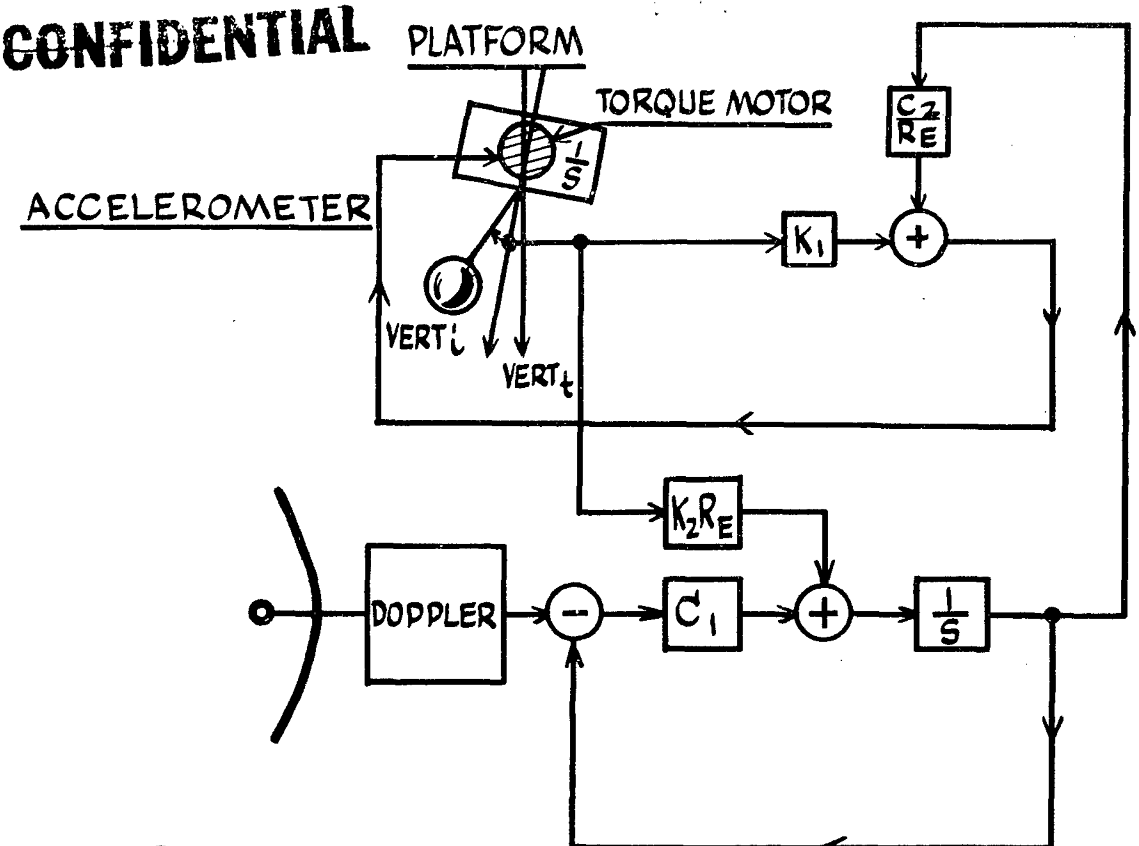

FIG. 6 a

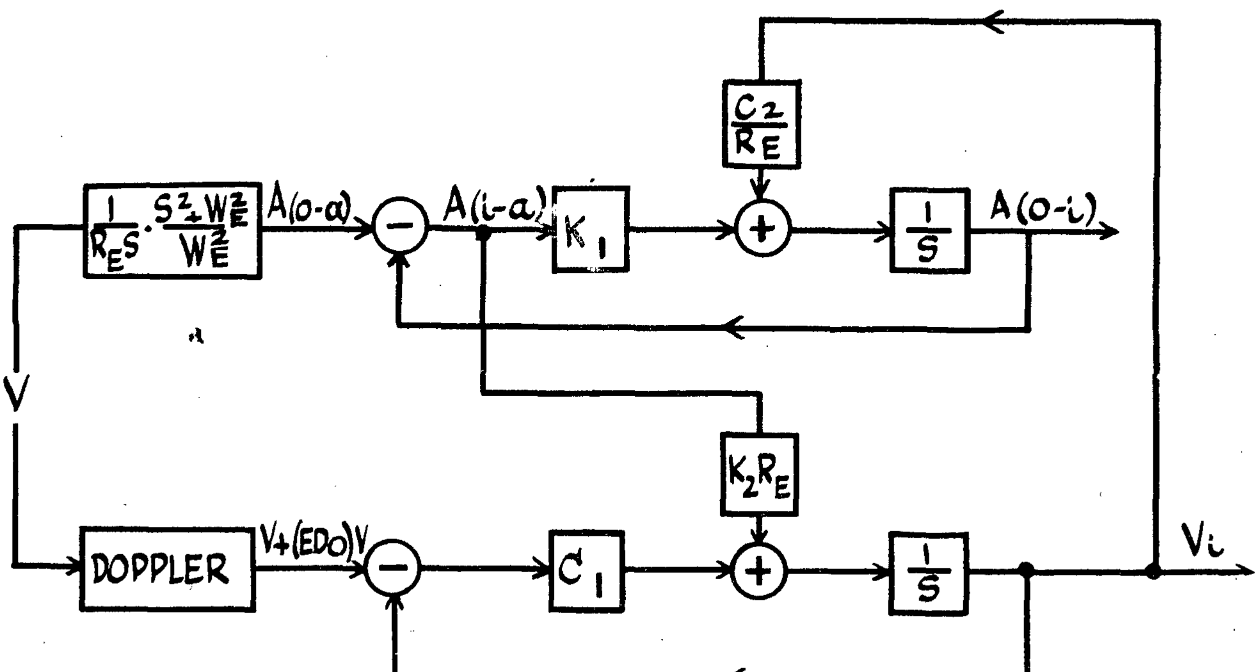

FIG. 6 b

FIG. 6 DOPPLER RADAR WITH 1st ORDER PLATFORM

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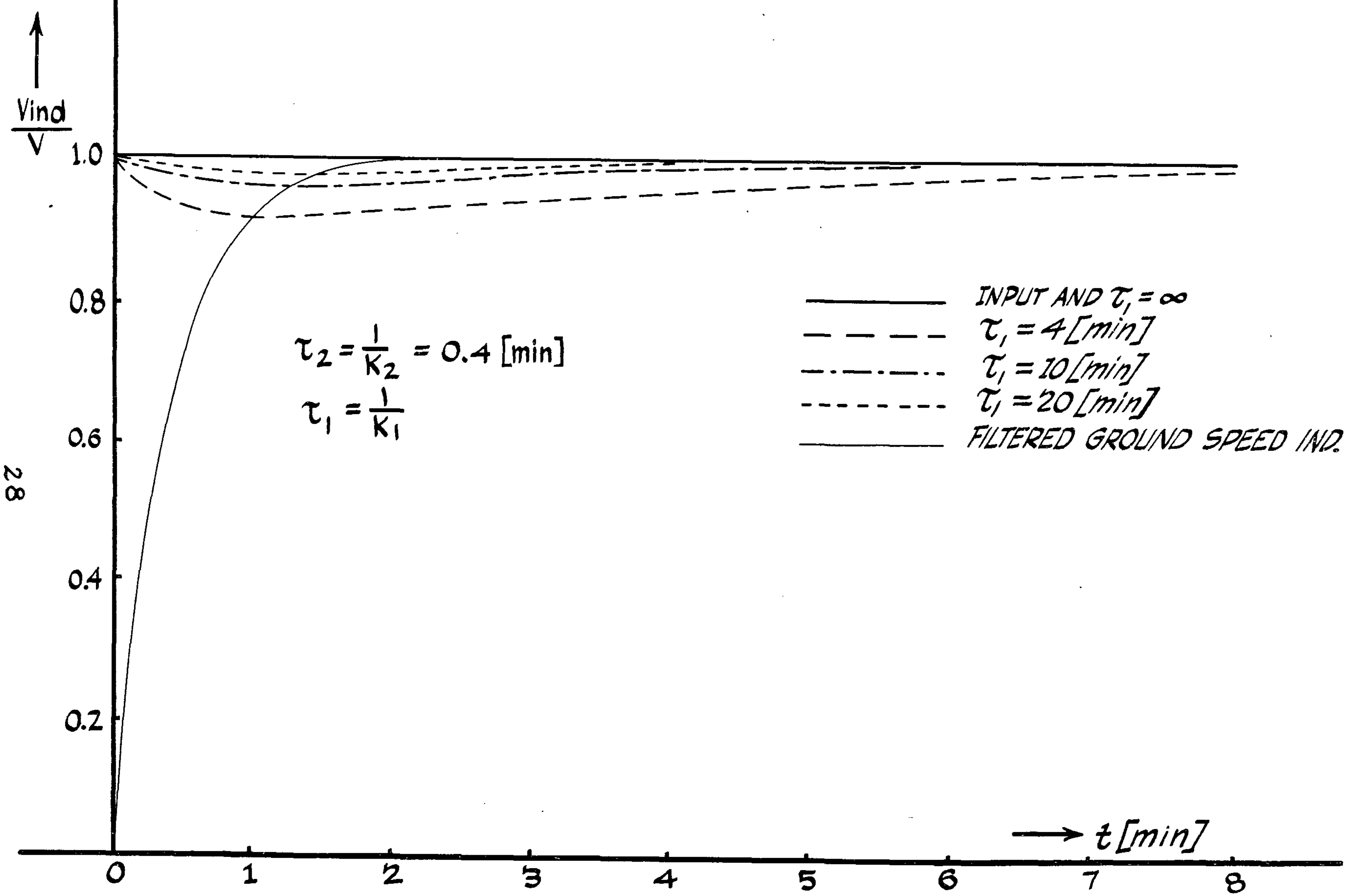

FIG. 7 SYSTEM RESPONSE TO UNIT STEP FUNCTION

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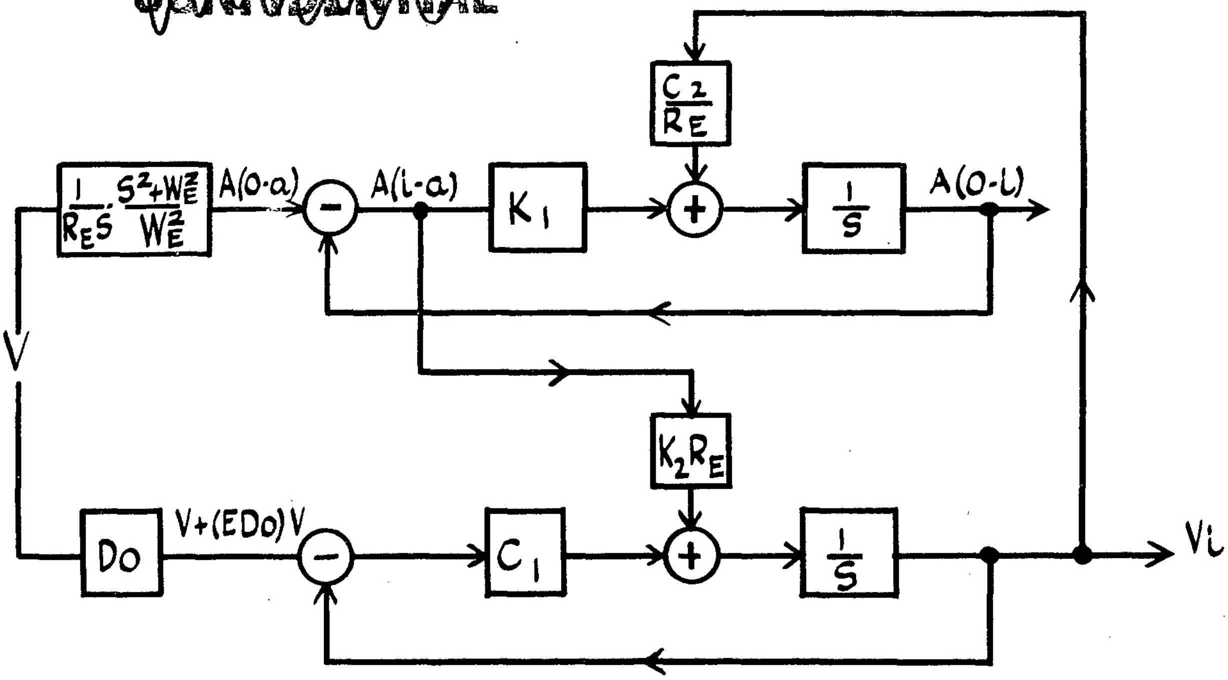

FIG. 8a (REPLOT OF 6b)

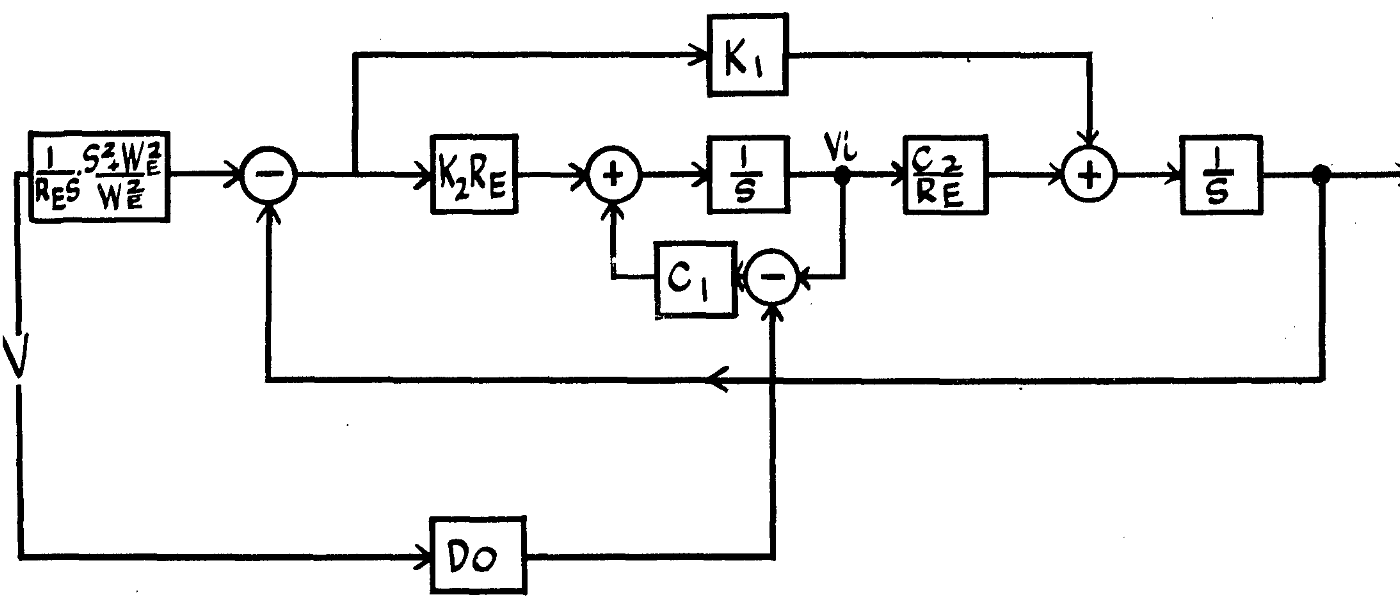

FIG. 8b (OTHER REPRESENTATION OF FIG. 8a)

FIG. 8 DOPPLER RADAR WITH 1st ORDER PLATFORM IN TWO DIFFERENT REPRESENTATIONS

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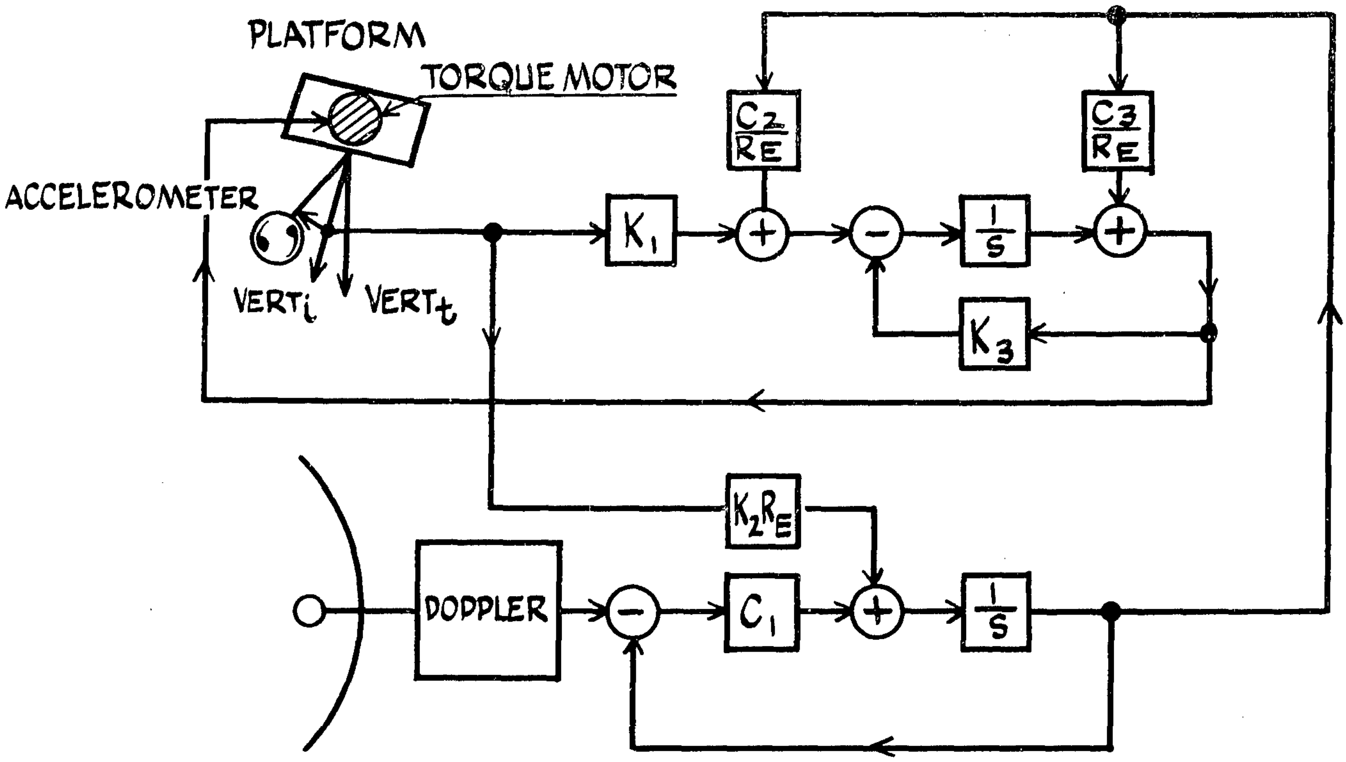

FIG. 9a

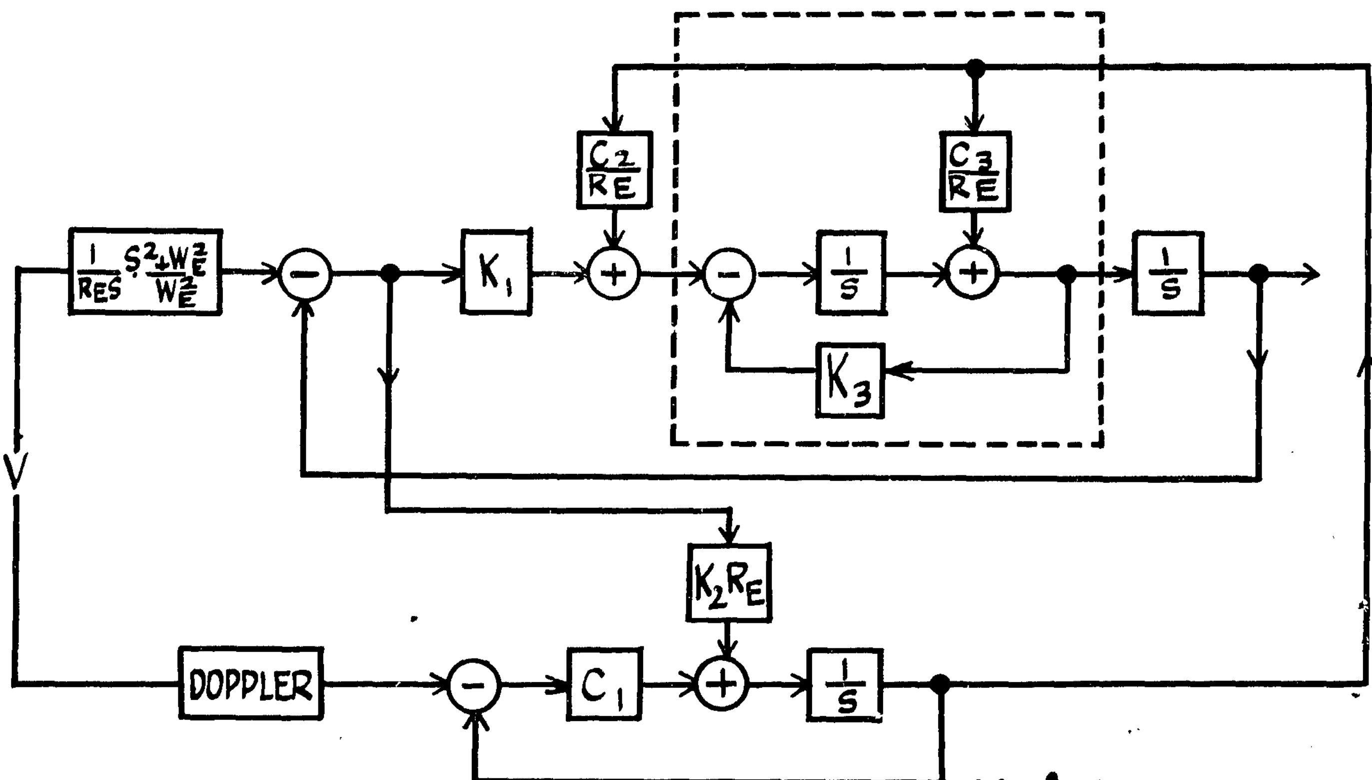

FIG. 9b

FIG. 9 DOPPLER RADAR WITH 2nd ORDER PLATFORM

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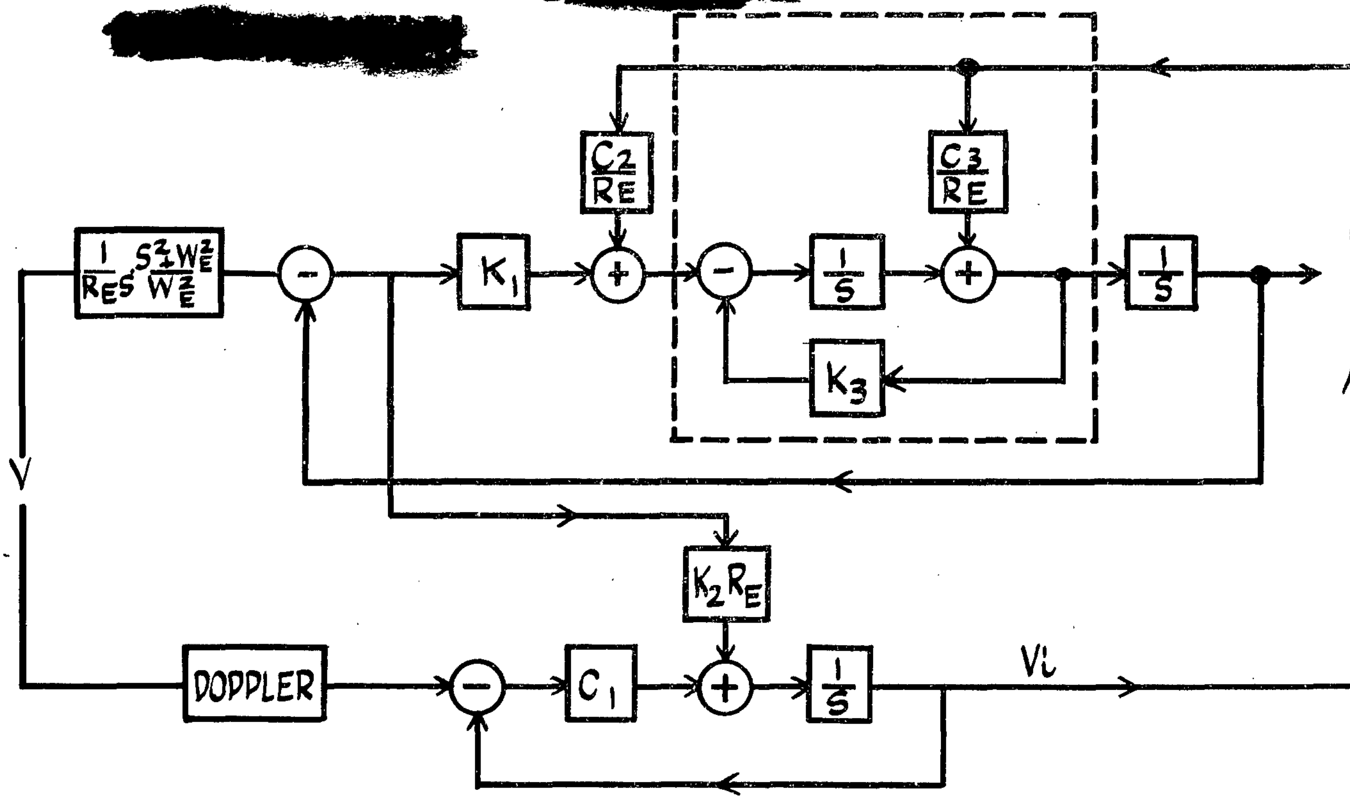

FIG. 10a (REPLOT OF FIG 9b)

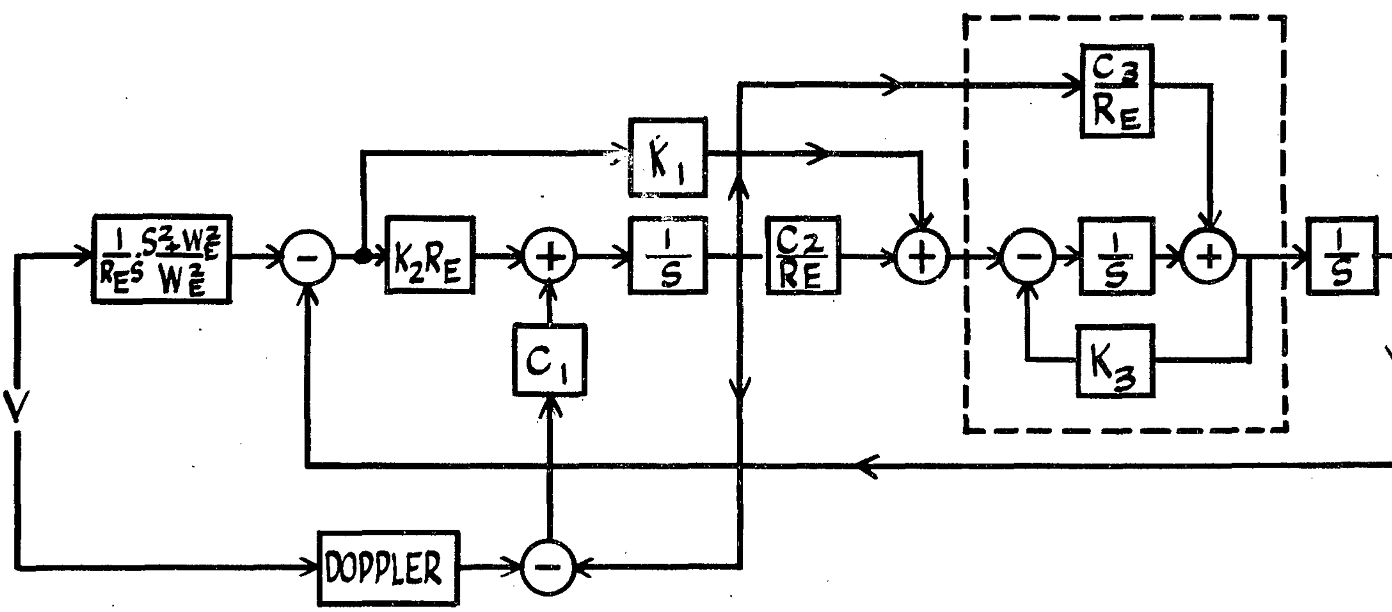

FIG. 10 b (OTHER REPRESENTATION OF FIG. 10a) ~~CONFIDENTIAL~~

FIG. 10 DOPPLER RADAR WITH 2nd ORDER PLATFORM IN TWO DIFFERENT REPRESENTATIONS

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